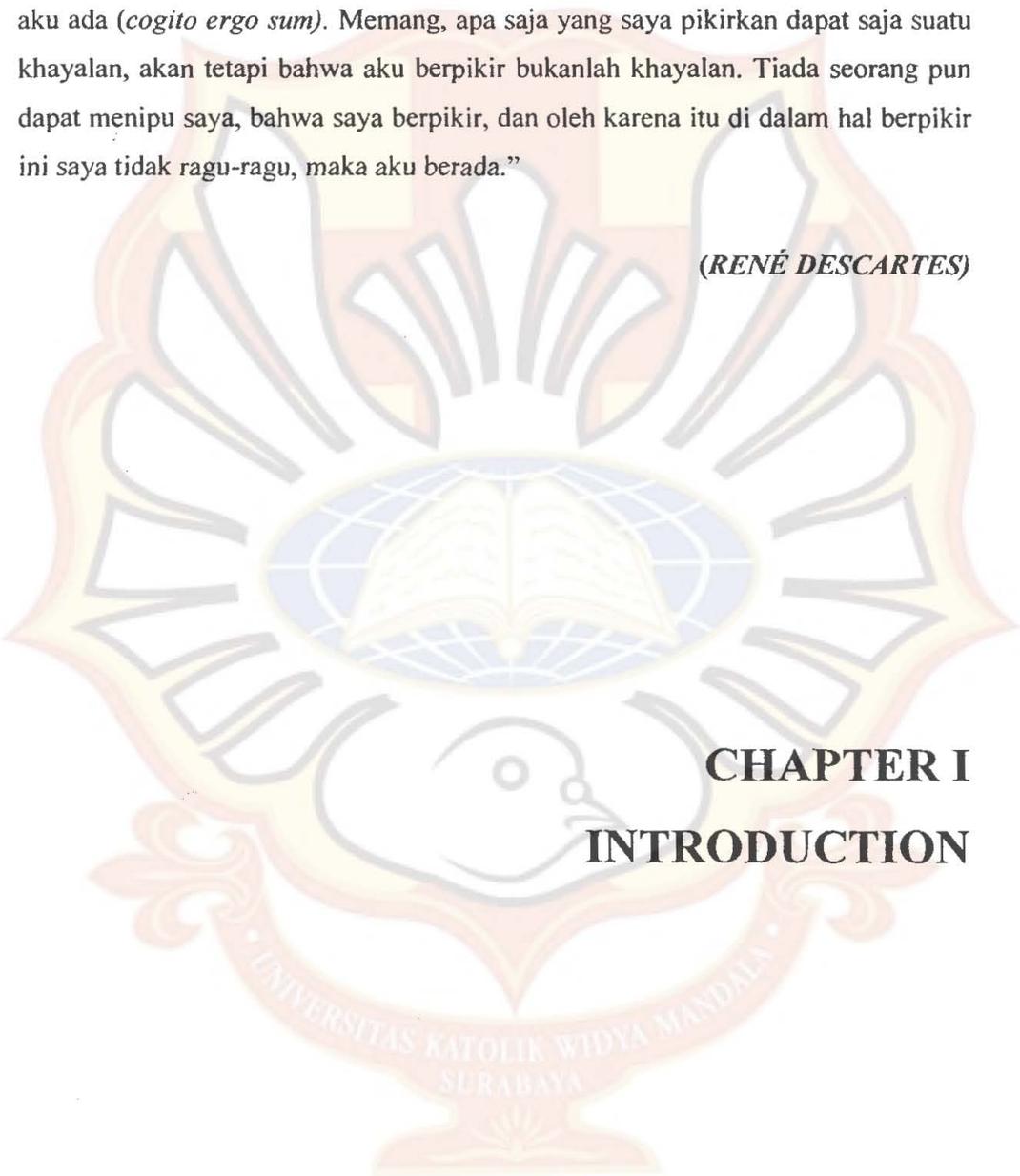


“Hanya ada satu hal yang tidak dapat diragukan. Mengenai satu hal ini tidak ada seorang pun yang dapat menipu kita, juga iblis tidak dapat, yaitu : bahwa aku ragu-ragu (aku meragukan segala sesuatu). Ini bukan khayalan, melainkan kenyataan. Aku ragu-ragu, atau aku berpikir dan oleh karena aku berpikir, maka aku ada (*cogito ergo sum*). Memang, apa saja yang saya pikirkan dapat saja suatu khayalan, akan tetapi bahwa aku berpikir bukanlah khayalan. Tiada seorang pun dapat menipu saya, bahwa saya berpikir, dan oleh karena itu di dalam hal berpikir ini saya tidak ragu-ragu, maka aku berada.”

(*RENÉ DESCARTES*)



## CHAPTER I INTRODUCTION

# CHAPTER I

## INTRODUCTION

### 1.1 Background

In typical industrial situations it is necessary to control  $p$  related quality characteristics of a process. One might consider handling this situation by apply  $p$  univariate control chart (Aparisi, 1996). However, it has long been realized that such an approach is unsatisfactory since it ignores the correlation between the characteristics and allows the overall rate different from the specified (Hayter and Tsui, 1994).

For the first time, Harold Hotelling developed the multivariate procedures in 1947. It is then recognized as the Hotelling's  $T^2$  control chart. Three criteria that must be possessed by multivariate control chart are (Houshmand and Golnabi, 1999; Hayter and Tsui, 1994) :

- a. The procedures gives a single answer to the question "Is the process in control ?"
- b. The specified type I error probability is properly maintained.
- c. The procedures must take into account the relationship between the variables.

The Hotelling's  $T^2$  control chart meets those criteria. Although the Hotelling's  $T^2$  control chart can accurately depict the out of control condition, it lacks the ability to detect which characteristic(s) is responsible for the out of control condition (Houshmand and Golnabi, 1999).

Several procedures that have been developed to interpret the out of control signal are (Firat, 2001; Mason, et al, 1997) :

1. Principal Component Analysis (PCA).
2. Step down procedure
3. Graphical technique
4. Mason, Young, Tracy (MYT) decomposition.

Another approach to interpret the out of control signal is to decompose the statistic value of the Hotelling's  $T^2$  control chart to its components which shows the contribution of each characteristic (Montgomery, 2001).

Anthony J. Hayter and Kwok Leung Tsui introduce the  $M$  control chart as an alternative of Hotelling's  $T^2$  control chart. The  $M$  control chart is a control chart which controls the first type error by finding the exact confidence interval. The confidence interval for the  $M$  control chart which involves more than two characteristics can be found by using simulation method. The  $M$  control chart uses the same confidence interval to control each characteristic involved. Therefore, the out of control signal interpretation becomes simpler. Although the  $M$  control chart is relatively simple, this procedure meets the three criteria that should be possessed by multivariate control chart (Hayter and Tsui, 1994).

The performance of the control chart can be measured by the Average Run Length (ARL). ARL is the average of the total observations needed before the control chart results in an out of control signal.

Two terms of ARL are  $ARL_0$  and  $ARL_1$ .  $ARL_0$  is the ARL which caused by the "false alarm". The "false alarm" is the situation where the control chart results in an out of control signal, while the process mean is not shifted.  $ARL_0$  is expected to be as large as possible.  $ARL_1$  is ARL which caused by the "true alarm". "True alarm" is the situation where the control chart results in an out of control signal because there is a shift in the process mean.  $ARL_1$  is expected to be as minimum as possible. The minimum value of the  $ARL_1$  is one (Kolarik, 1995).

This thesis studies the effect of changes in covariance structure to the performance of the Hotelling's  $T^2$  and  $M$  control chart. It also includes the effectiveness of out of control signal interpretation method from both control charts.

## 1.2 Problem Statements

The problem statement of this thesis are :

- a. What are the effects of changes in the covariance structure to the performance of Hotelling's  $T^2$  and  $M$  control chart ?

- b. What are the effects of changes in the covariance structure to the effectiveness of the out of control signal interpretation method for the Hotelling's  $T^2$  and  $M$  control chart ?

### 1.3 Problem Scope

The problem scope of this thesis are :

- a. The performance and the effectiveness of out of control signal interpretation method on both control charts are resulted from simulation approach.
- b. The performance measure to be applied on both control charts is ARL.
- c. Data for simulation are generated by software S-PLUS 2000 for Windows.
- d. The number of characteristics is three.
- e. Data are normally distributed with a mean of zero and variance of one.
- f. The covariance of each pair of characteristics is the combination of :
  1. -0.3, which represents a weak and negative relationship.
  2. -0.5, which represents a moderate and negative relationship.
  3. -0.8, which represents a strong and negative relationship.
  4. 0.3, which represents a weak and positive relationship.
  5. 0.5, which represents a moderate and positive relationship.
  6. 0.8, which represents a strong and positive relationship.
- g. The sample size of each observation is one (individual observation).
- h. Statistic that is to be considered in this thesis is only the mean of the process. The mean shifts are the combination of  $0\sigma$ ,  $0.5\sigma$ ,  $1\sigma$ ,  $1.5\sigma$ ,  $2\sigma$ , and  $2.5\sigma$ .
- i. The Hotelling's  $T^2$  out of control signal interpretation method in this thesis uses the decomposition method (Montgomery, 2001).

### 1.4 Objectives

To know what is the effect of changes in covariance structure to the performance of Hotelling's  $T^2$  and  $M$  control chart with theirs out of control signal interpretation method, respectively.

### 1.5 Assumptions

The assumptions used in this thesis are :

- a. The weight for each characteristic is equal.
- b. The out of control signal interpretation method for Hotelling's  $T^2$  control chart is written as Montgomery's method for the rest of this thesis. The out of control signal interpretation method for  $M$  control chart is written as Hayter and Tsui's method for the rest of this thesis.
- c. The number of replication is 1000 times.
- d. The value of the TRUE condition is one (1) while the value of the FALSE condition is zero (0).
- e. The term of characteristic has the same meaning with the term of variable.
- f. The effect of mean shift in any characteristic is equal if the power of mean shift is equal.
- g. Mean shift that below two standard deviation will be regarded as small mean shift, while mean shift that greater than or equal to two standard deviation will be regarded as large mean shift.
- h. Since the number of characteristics involved is three and the covariance matrix is symmetric, then for simplicity and readability, the covariance structure is displayed only by the three elements within covariance structure. For example the covariance matrix :

$$\Sigma = \begin{bmatrix} 1 & -0.5 & -0.3 \\ -0.5 & 1 & 0.8 \\ -0.3 & 0.8 & 1 \end{bmatrix}$$

is displayed with a vector (-0.5, -0.3, 0.8). It means that the covariance between first and second characteristic is -0.5, first and third characteristic is -0.3, and second and third characteristic is 0.8.

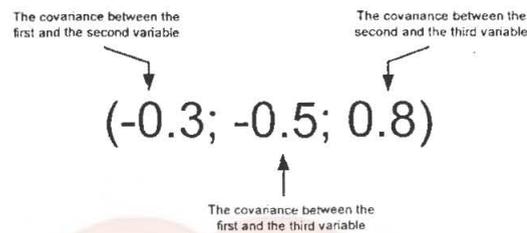


Figure 1 : Vector that Represents Covariance Matrix

## 1.6 Organization

The organization of this thesis is :

### Chapter I : Introduction

This chapter consists of the background, problem statements, problem scope, the objectives, used assumptions, and organization.

### Chapter II : Literature Review

This chapter consists of the theories which support the research in this thesis. This chapter also consists of the procedure to solve the problems.

### Chapter III : Research Methodology

This chapter consists of explanation about the procedure used in the research of this thesis.

### Chapter IV : Data

This chapter consists of data that have been generated by computer simulation. The simulation results are categorized into several groups to support the analysis.

### Chapter V : Analysis of Simulation Results

This chapter consists of the analysis of the simulation results that aimed to answer the problem statement.

## Chapter VI : Conclusions and Suggestions

This chapter consists of the conclusions that answer the problem statement based on the simulation results and suggestions for further research.

