

THESIS

"THE EFFECTS OF CHANGES IN COVARIANCE STRUCTURE TO THE PERFORMANCE OF THE HOTELLING'S T^2 AND M CONTROL CHART"



COMPOSED BY :

RIYANTO WIBOWO

(5303000005)

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Satu (1)

**INDUSTRIAL ENGINEERING DEPARTMENT
ENGINEERING FACULTY
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APPROVAL

Thesis entitled “**THE EFFECT OF CHANGES IN COVARIANCE STRUCTURE TO THE PERFORMANCE OF HOTELLING’S T^2 AND M CONTROL CHART**” which submitted by

Name : Riyanto Wibowo

NRP : 5303000005

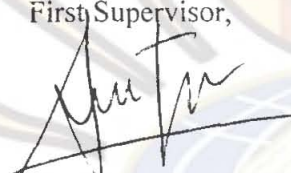
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Surabaya, September 3rd 2004

Board of Supervisor,

First Supervisor,

Second Supervisor,


(Drs. Peter R. Angka, M.Kom)

NIK : 511.88.0136



(Hendry Raharjo, ST)

NIK : 531.01.0589

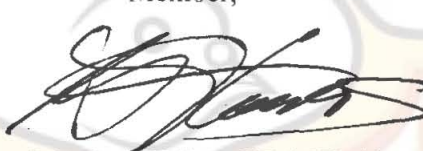
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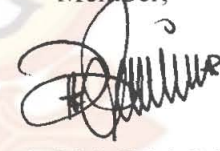
Member,


(Dian Retno SD, ST, MT)

NIK : 531.97.0298


(Martinus Edi S., ST, MT)

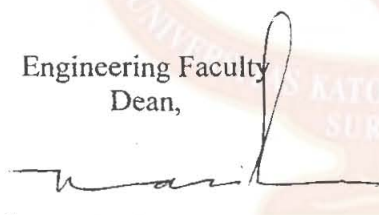
NIK : 531.98.0305


(Anastasia LM, ST, MMT, M.Sc)

NIK : 531.03.0564

Engineering Faculty
Dean,

Industrial Engineering Department
Chairman,


(Ir. Nani Indraswati, M.Sc)

NIK : 521.86.0121


(Dian Retno SD, ST, MT)

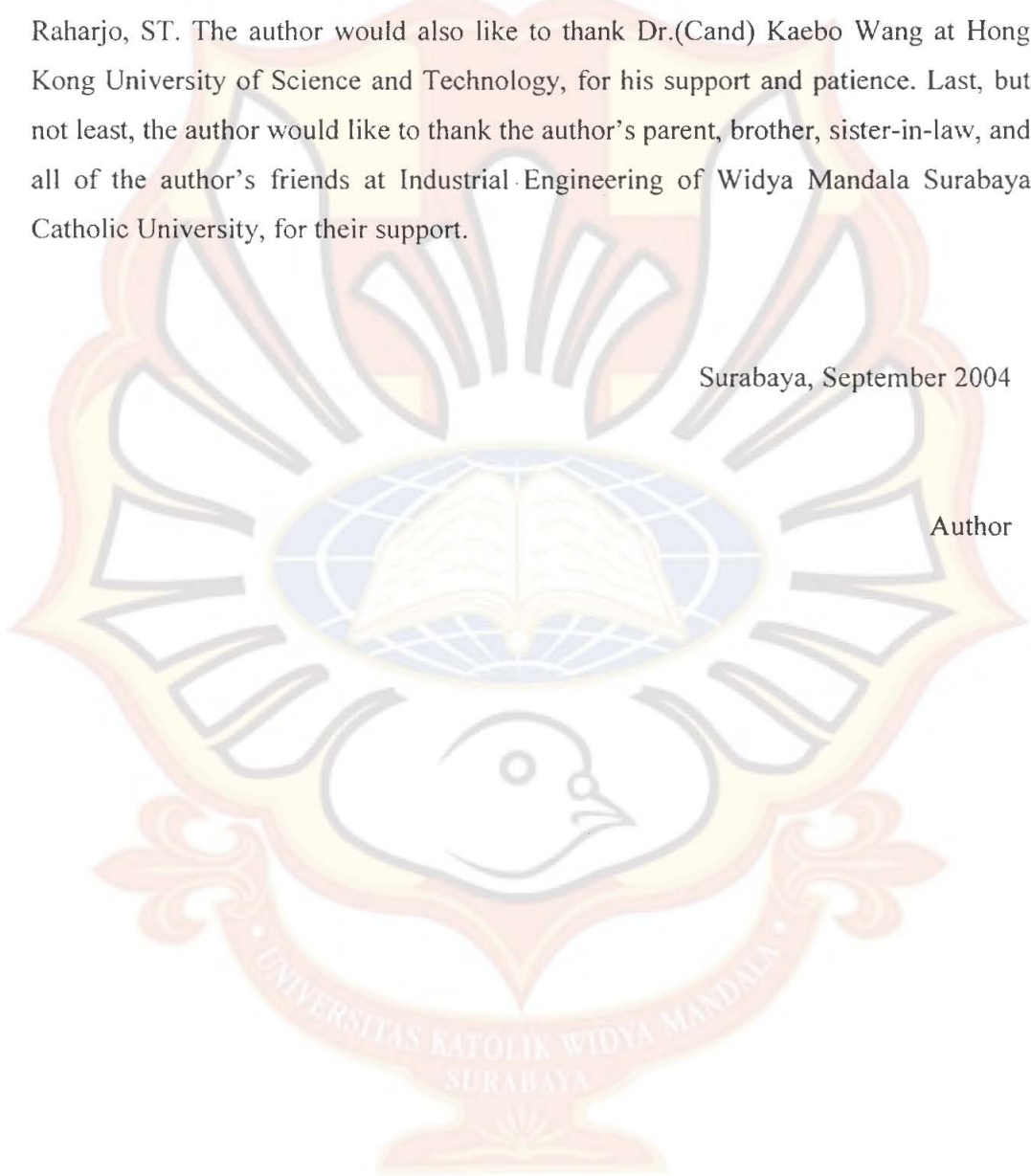
NIK : 531.97.0298

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ABSTRACT

Multivariate control chart is used for control several related characteristics simultaneously. Since the covariance structure measures the linear association between two characteristics, multivariate control procedures should consider the covariance structures among the controlled characteristics. One of the persistent problems in multivariate control chart procedures is the interpretation of out of control signal, that is, to interpret which characteristic(s) responsible for the signal. This thesis analyzes the effects of changes in covariance structure to the performance of Hotelling's T^2 and M control charts, with their interpretation methods respectively. The simulation results show that the performance of Hotelling's T^2 control chart is affected by the changes in covariance structure, while the performance of M control chart is not. The simulation results also show that the performance of Hotelling's T^2 control chart is better than the performance of M control chart.

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LIST OF NOTATIONS

p = number of characteristics (variables)

n = sample size

m = replication number

i = replication variable

j = sample size variable

k = characteristic variable

\bar{x} = sample mean

μ = process mean (target)

X = observation matrix

Σ = population covariance matrix

Σ^{-1} = inverse of covariance matrix

r = correlation coefficient

R = correlation coefficient matrix

T^2 = statistic of Hotelling's control chart

$T_{(k)}^2$ = statistic of Hotelling's control chart without k th variable

d_k = the k th statistic of the Montgomery's method

M = statistic of M control chart

$C_{R,\alpha}$ = critical region with type I error (α)

V = method condition (contains TRUE or FALSE)

E = method efficiency

$'$ = matrix transpose operation

λ = eigenvalue